საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 13, №4, 2019
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 13, no. 4, 2019

Mechanics

Construction of the Main Transverse-Vertical Cross-Sections for Orthopedic Shoe Trees and 3D Design of the Shoe Tree Frame

Merab Shalamberidze*, Zaza Sokhadze, Malvina Tatvidze§**

* *Department of Design and Technology, Akaki Tsereteli State University, Kutaisi, Georgia*
** *Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia*
§ *Department of Chemical Technology, Akaki Tsereteli State University, Kutaisi, Georgia*

(Presented by Academy Member Guram Gabrichidze)

ABSTRACT. The shoe trees for production of individual orthopedic shoes should be designed with maximum consideration of pathologies of the deformed feet. For normal functioning of the deformed feet it is necessary to design the orthopedic shoes with the special shape inside allowing the patients to feel comfortable while wearing them. In the present paper the methods of construction of the geometric forms of the shoe tree main cross-sections and the problems of 3D design of the shoe tree frame are considered. The research is mainly based on the patients' database providing anthropometric, pedographic and tensometric data of the deformed and pathological feet. Integral curves of the singular boundary Dirichlet problem solutions were used to construct the main cross-sections of the orthopedic shoe trees. The sections of the obtained curves were joined and rotated with the help of the computer programs, on the basis of which the shapes of transverse-vertical cross-sections of the orthopedic shoe trees were constructed in 0.3D and 0.07D. The orthopedic shoe tree is constructed in spatial format by means of the main transverse-vertical cross-sections and 3D design method (Delcam). *© 2019 Bull. Georg. Natl. Acad. Sci.*

Key words: orthopedic shoe tree, differential equations, integral curves

In the manufacture of special-purpose shoes it is important to focus on 3D design of orthopedic shoe trees. From the geometrical point of view the shoe tree has a complex shape. The description of shoe tree by means of mathematical methods is a very time-consuming and labor-intensive process. Generally, there are different technical aspects of the shoe tree design. In the process of shoe tree design the foot size and shape should be taken into consideration.

The algorithms describing the geometrical shape of the shoe tree surface are considered in some scientific works [1-6], where for description of the geometrical shape of the shoe tree the following mathematical methods are used: Radial graphical, Bi-quadratic spline, Bi-cubic interpolar spline. The mentioned methods are quite difficult and labor-intensive requiring a great deal of time for designing the shoe trees.

We carried out anthropometric, tensometric and pedographic surveys of the deformed and pathologic feet and created a database of the patients with specific pathological abnormalities of feet. The obtained parameters are transformed according to the bio-mechanical characteristics of the human feet movement and based on that the curved lines describing the shoe tree surface are defined. We developed the shapes of the main transversal-vertical cross-sections of orthopedic shoe tree, and after that we developed the 3D design of the shoe tree and constructed the spatial frame. Development of a new algorithm describing the geometrical surface of a shoe tree and its application in practice is one of the most relevant and topical problems in the manufacture of individual orthopedic shoes.

For systemic description of the main transversal-vertical cross-sections of the orthopedic shoe trees the integral curves of singular boundary Dirichlet problem solutions are studied and applied. The shapes of the transverse-vertical cross-sections of the orthopedic shoe tree obtained by means of the integral curves of the Dirichlet singular boundary problem solutions represent important novelty in the present research. The solution of the problem will take comparatively less time and the result obtained will be more precise.

In [7] I. Rakunkova, A. Spieleuer, S. Stanek and E. Weinmüller consider the singular boundary Dirichlet problem:

$$u''(t)+\frac{a}{t}u'(t)-\frac{a}{t^2}u(t)=f(t,u(t),u'(t)) \tag{1}$$

$$u(t)=0, \quad u(T)=0(2), \tag{2}$$

where $a\in(-\infty;1)$, f satisfies the Caratheodory local condition for the set of $[0,T]\times D;\ \ D=(0;+\infty)\times R.$

In the mentioned work the question of existence of the solution of problems (1) - (2) and also, in Lemma 3.1 of the same work the solution of problems (1) - (2) are explicitly given, in particular:

$$u(t)=c_1t+c_2t^{-a}+t\int_t^T S^{-a-2}\left(\int_S^T \xi^{a+1}f(t,u(\xi),u'(\xi)d\xi\right)d\xi, \tag{3}$$

where $C_1C_2\in\pi t\in[0,T]$.

Our goal is clearly to write the solution for different cases of function $f(t,u(t),u'(t))$ in the right-hand side part of equation (1), as well as for different values of a using formula (3), and then to construct the integral curves of those solutions allowing us to get the desired shapes of the transverse-vertical cross-sections for orthopedic shoe tree.

Consider the first private case of problems (1) - (2):

$$u''+\frac{2}{t}u'-\frac{2}{t^2}u=t \tag{4}$$

$$u(1)=0\,,\ u'(1)=c\,, \tag{5}$$

i.e. taking into consideration $f\left(t,u(t),u'(t)\right)=t,\ \ a=-2,\ \ t\in[0,1],$ the solution of the problems (4)-(5) is given by:

$$u(t)=\frac{t^2}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}.$$

If c=0, then $u(t)=\frac{t^2}{2}-t+\frac{1}{2}.$

Consider the second private case of the same (1) - (2) problems:

$$u''+\frac{a}{t}u'-\frac{a}{t^2}u=t^2 \tag{6}$$

$$u(1)=0\,,\ u'(1)=c\,, \tag{7}$$

i.e. taking into consideration $f\left(t,u(t),u'(t)\right)=t,\quad a=-2,\ \ t\in\left[0,1\right],$ the solution of the problems (6)-(7) is given by:

$$u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t-\frac{t^2}{2}+\frac{t^4}{6}.$$

If c=0, then $u(t)=\frac{t^4}{6}-\frac{t^2}{2}+\frac{1}{3}t.$

Using the above-mentioned method we constructed shapes of transverse-vertical cross section of individual orthopedic shoe trees 0.18D; 0.4D; 0.5D; 0.62D; 0.72D; 0.8D and 0.9D (where D is the foot length) [8-12].

Thus, by means of integral curves of the solutions of the above differential equations we constructed the shapes of the main transverse-vertical cross-section for individual orthopedic shoe trees in 0.3D and 0.07D.

For construction of a shape of the transverse-vertical cross-section for individual orthopedic shoe tree in 0.3D we divided it into nine pieces in advance and described each numbered section by means of the integral curves of solution of differential equations given below. Out of the integral curves we chose nine parts identical to the geometric shapes of the transverse-vertical cross-section of orthopedic shoe tree in 0. 3D, namely:

1. AB curve corresponds to that part of the solution of equation $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, for which c = 2 and corresponds to the set of [1.5; 2.8]× [2.9; 14.4];
2. BC curve corresponds to that part of the solution of equation $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, for which c = 0 and corresponds to the set of [0.2; -1.5]× [0.5; 0.55];
3. CD curve corresponds to that part of the solution of equation $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, for which c = 2 and corresponds to the set of [-2.3; -2.8]× [- 8.6; -13];
4. DE curve corresponds to that part of the solution of equation $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t-\frac{t^2}{2}+\frac{t^4}{6}$, for which c = 2 and corresponds to the set of [-1.8; -3.8]×[2.1; 9.8];
5. EF curve corresponds to that part of the solution of equation $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t-\frac{t^2}{2}+\frac{t^4}{6}$, for which c = 0 and corresponds to the set of [2.8; 3.0]×[9.8; 13,8];
6. FM curve corresponds to that part of the solution of equation $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t-\frac{t^2}{2}+\frac{t^4}{6}$, for which c = 0 and corresponds to the set of [-3.1; -3.4]×[8.5; 13.8];
7. MN curve corresponds to that part of the solution of equation $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, for which c = 0 and corresponds to the set of [2.5; 1.95]× [6.05; 2.5];

8. NP curve corresponds to that part of the solution of equation $u(t) = (-\frac{1}{3} - \frac{1}{3}c)t + \frac{1}{3}c\frac{1}{t^2} + \frac{2}{3}t - \frac{t^2}{2} + \frac{t^4}{6}$, for which c = 0 and corresponds to the set of [1.8; -1]×[0; -0.6];

9. PA curve corresponds to that part of the solution of equation $u(t) = \frac{t^3}{2} - \frac{1}{3}ct^{-2} - \left(1 - \frac{1}{3}c\right)t + \frac{1}{2}$, for which c=2 and corresponds to the set of [-1.7; -2.05]×[- 2.6; -5.9].

Using the computer program, we rotated and joined the mentioned lines, on the basis which we obtained the shape of transverse-vertical section of the orthopedic shoe tree in 0.3D (Fig. 1).

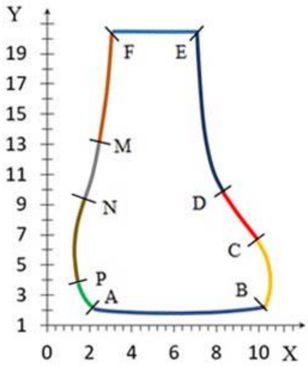

Fig. 1. The shape of transverse-vertical section of the orthopedic shoe tree in 0.3D.

The transverse-vertical section of the orthopedic shoe tree was constructed in the same way in 0.07D. For construction of the shape of individual orthopedic shoe tree of a transverse-vertical cross-section in 0.07D we divided it into eight parts in advance and described each numbered section by means of the integral curves of solution of differential equations given below. Out of the integral curves we chose the eight parts, which are identical to the shapes of the transverse-vertical section of orthopedic shoe trees in 0.07D, namely:

1. AB curve corresponds to that part of the solution of equation $u(t) = (-\frac{1}{3} - \frac{1}{3}c)t + \frac{1}{3}c\frac{1}{t^2} + \frac{2}{3}t - \frac{t^2}{2} + \frac{t^4}{6}$, for which c = 0 and corresponds to the set of [3.1; 2.9]×[10.1; 13.8];

2. BC curve corresponds to that part of the solution of equation $u(t) = \frac{t^3}{2} - \frac{1}{3}ct^{-2} - \left(1 - \frac{1}{3}c\right)t + \frac{1}{2}$, for which c = 2 and corresponds to the set of [-12.8; -3.1]× [- 0.3; -0.8];

3. CD curve corresponds to that part of the solution of equation $u(t) = \frac{t^3}{2} - \frac{1}{3}ct^{-2} - \left(1 - \frac{1}{3}c\right)t + \frac{1}{2}$, for which c = 2 and corresponds to the set of [-1.8; -2.5]× [- 4.1; -7.8];

4. DE curve corresponds to that part of the solution of equation $u(t) = (-\frac{1}{3} - \frac{1}{3}c)t + \frac{1}{3}c\frac{1}{t^2} + \frac{2}{3}t - \frac{t^2}{2} + \frac{t^4}{6}$, for which c = 1 and corresponds to the set of [-0.5; -0.85]×[3.6; 0,1];

5. EF curve corresponds to that part of the solution of equation $u(t) = \frac{t^3}{2} - \frac{1}{3}ct^{-2} - \left(1 - \frac{1}{3}c\right)t + \frac{1}{2}$, for which c = 2 and corresponds to the set of [0.5; 0.7]×[- 12.8; -3,1];

6. FM curve corresponds to that part of the solution of equation $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, for which c = 0 and corresponds to the set of [-2.05; -1.1]× [- 1.6; 1.0];

7. MN curve corresponds to that part of the solution of equation $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t-\frac{t^2}{2}+\frac{t^4}{6}$, for which c = 3 and corresponds to the set of [-1.7; -2.3]×[2.85; 6.2];

8. NA curve corresponds to that part of the solution of equation $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t-\frac{t^2}{2}+\frac{t^4}{6}$, for which c = 5 and corresponds to the set of [0.8; 0.4]×[- 7.95; 1.1].

The shape of the transverse-vertical section of the individual orthopedic shoe tree in 0.07D is shown in Fig. 2.

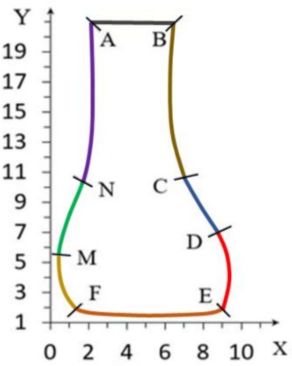

Fig. 2. The shape of transverse-vertical section of the orthopedic shoe tree in 0.07D.

By means of the shapes of the main transverse-vertical sections of individual orthopedic shoe tree and the 3D design program (Delcam) we constructed the spatial frame for the shoe tree (Fig. 3).

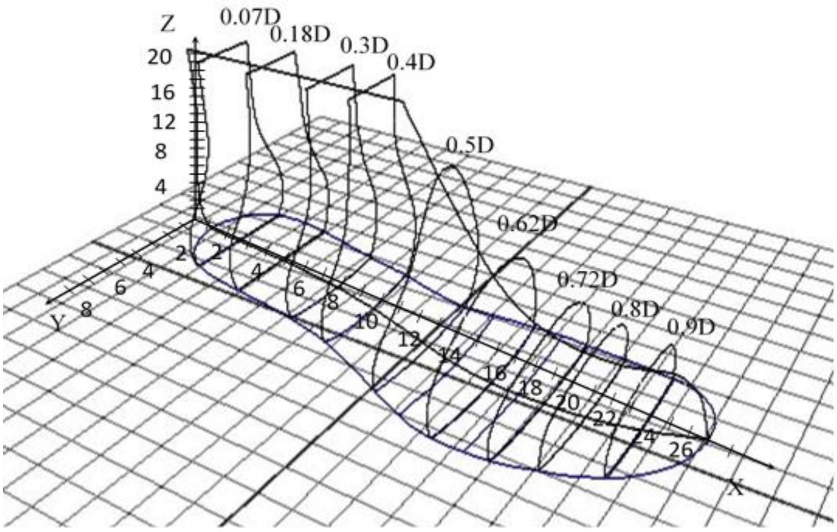

Fig. 3. The frame for the individual orthopedic shoe tree in 3D format.

Thus, based on the patients’ database and the integral curves of the solutions of the Dirichlet singular boundary problem we constructed the shapes of the main transverse-vertical sections for orthopedic shoe trees in 0.3D and 0.07D and the spatial frame of the shoe tree by the use of 3D design program (Delcam). The above method of mathematical research permits precisely to describe the shapes of main sections of the orthopedic shoe tree. Besides, it allows to change the shapes of transverse-vertical sections all the time. The latter is particularly relevant in the manufacture of orthopedic shoes for the patients with deformed and pathological feet.

The work was fulfilled with the financial support of Shota Rustaveli National Science Foundation of Georgia, Grant FR № 217386.

მექანიკა

ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი განივ-ვერტიკალური კვეთების აგება და კალაპოტის კარკასის 3D პროექტირება

მ. შალამბერიძე*, ზ. სოხაძე, მ. თათვიძე§**

** აკაკი წერეთლის სახელმწიფო უნივერსიტეტი, დიზაინისა და ტექნოლოგიის დეპარტამენტი, ქუთაისი, საქართველო*
*** აკაკი წერეთლის სახელმწიფო უნივერსიტეტი, მათემატიკის დეპარტამენტი, ქუთაისი, საქართველო*
§ აკაკი წერეთლის სახელმწიფო უნივერსიტეტი, ქიმიური ტექნოლოგიის დეპარტამენტი, ქუთაისი, საქართველო

(წარმოდგენილია აკადემიის წევრის გ. გაბრიჩიძის მიერ)

ინდივიდუალური ორთოპედიული ფეხსაცმლის დასამზადებლად საჭიროა ისეთი კალაპოტების პროექტირება, სადაც მაქსიმალურად იქნება გათვალისწინებული დეფორმირებული ტერფების პათოლოგიური გადახრები. დეფორმირებული ტერფის ნორმალური ფუნქციონირებისათვის საჭიროა შევიმუშაოთ სპეციალური ორთოპედიული ფეხსაცმლის ისეთი შიგა ფორმა, რომლის ტარების პროცესში პაციენტი თავს იგრძნობს კომფორტულად. სტატიაში განხილულია ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი კვეთების გეომეტრიული ფორმების აგების მეთოდები და კალაპოტის კარკასის 3D პროექტირების საკითხები. კვლევის პროცესში სტატიის ავტორთა ჯგუფი ძირითადად ეყრდნობოდა პაციენტთა მონაცემთა ბაზას, რომელშიც აღწერილია დეფორმირებული და პათოლოგიური ტერფების ანთროპომეტრიული, პედოგრაფიული და ტენზომეტრული მონაცემები. ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი კვეთების ფორმების ასაგებად გამოყენებულია დირიხლეს სინგულარული სასაზღვრო ამოცანის ამონახსნთა ინტეგრალური წირები. კომპიუტერული პროგრამების საშუალებით ვაწარმოებდით მიღებული წირების

მონაკვეთების შეუღლებასა და გადაბმას, რის საფუძველზეც აგებულია ორთოპედიული ფეხსაცმლის კალაპოტის განივ-ვერტიკალური კვეთის ფორმები 0,3D-ზე და 0,07D-ზე. ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი განივ-ვერტიკალური კვეთების საშუალებითა და 3D პროექტირების მეთოდის გამოყენებით (Delcam) აგებულია კალაპოტის კარკასი სივრცულ ფორმატში.

REFERENCES

1. Fukin V.A. (1980) Radiusograficheskii metod konstruirovaniia vnutrennei formy obuvi, 381 s. M. (in Russian).
2. Fukin V.A., Kostyleva V.V., Lyba V.P. (1987) Proektirovanie obuvnykh kolodok, 268 s. M. (in Russian).
3. Fukin V.A. (2000) Teoreticheskie osnovy proektirovaniia vnutrennei formy obuvi. 2-izdanie per. i dop. s. 356. M. (in Russian).
4. Kiselev S.Yu. (2004) Avtomatizirovannoe proektirovanie i izgotovlenie tekhnologicheskoi osnastki dlia proizvodstva obuvi i protezo-ortopedicheskikh izdelii. Diss. d-ra tekh. nauk., 392 s. M. (in Russian).
5. Kniagicheva N.V., Golovanov S.A., Kiselev S.Yu., Shevchenko A.V. (2016) Primenenie 3d-skanirovaniia pri provedenii antropometricheskikh issledovanii stop. *Zhurnal dizain i tekhnologii.* **53**(95):31-39. M. (in Russian).
6. Zamarashkin K.N. (2004) Matematicheskie metody proektirovaniia obuvi i konstruirovaniia tekhnologicheskoi osnastki. s. 312. M. (in Russian).
7. Rachunkova I., Spielaurer A., Stanek S. and Weinmuler E. B. (2013) The structure of a set of positive solutions to Dirichlet BVPs with time and space singularities. *Georgian Mathematical Journal.* 1: 95-127.
8. Shalamberidze M., Sokhadze Z., Tatvidze M. (2018) Construction of the transverse-vertical shapes of the orthopedic boot tree by means of the solution to singular Dirichlet boundary value problem. *Bull. Georg. Natl. Acad. Sci.,* **12**,1:27-32.
9. Shalamberidze M., Sokhadze Z., Tatvidze M. (2018) Construction of the orthopedic shoe tree main transverse-vertical cross-sections by means of the integral curves. *Bull. Georg. Natl. Acad. Sci.,* **12**,3:23-30.
10. Shalamberidze M., Sokhadze Z. (2018) Constructing a shape of orthopedic boot-tree print by means of the solution to differential equation with deviating argument. *International Scientific Journal Theoretical & Applied Science,* **61**,5:27-32. Philadelphia USA.
11. Shalamberidze M., Sokhadze Z., Lomtadze N. (2019) Constructing the transverse-vertical geometrical shape of the orthopedic shoe-tree in section 0.5d using the solutions of singular Dirichlet boundary problems. *International Scientific Journal Theoretical & Applied Science,* **74**,6:501-504. Philadelphia USA.
12. Shalamberidze M., Sokhadze Z., Tatvidze M. (2019) Construction of the orthopedic boot tree print and main longitudinal vertical section by means of the solution of differential equations. *Bull. Georg. Natl. Acad. Sci.,* **13**,2:17-21.

Received May, 2019