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Mechanics

Construction of the Orthopedic Boot Tree Print and Main Longitudinal Vertical Section by Means of the Solution of Differential Equations

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ABSTRACT. The methods for construction of the main longitudinal-vertical section, print and computer design of orthopedic boot tree are considered in the paper. The research is based on anthropometric, pedographic and tensometric data of the patients with club feet. To manufacture orthopedic shoe, it is necessary to design boot tree taking into account pathological deviation of the club foot. For normal functioning of the club foot an internal shape of the shoe is developed allowing patient to feel comfortable. Geometrically, orthopedic boot tree has a complex shape and its description by the methods of mathematical investigation is a long and arduous process. The integral curves of the solutions of singular Dirichlet boundary value problem and the equations with deviating argument are used to construct the main longitudinal-vertical section and the print of the orthopedic boot tree. By means of a computer program we executed turning and connection of the obtained curves, on the basis of which we constructed the main longitudinal-vertical section and print of orthopedic boot tree. The main longitudinal-vertical section and the print of orthopedic boot tree in 3D format are described in the paper. *© 2019 Bull. Georg. Natl. Acad. Sci.*

Key words: orthopedic boot tree, integral curves of the solutions to differential equations

In the orthopedic shoe industry, considerable attention is paid to design and production of special-purpose boot trees. It is well-known that geometrically, the orthopedic boot tree has complex shape. Its description by means of the solution curves of differential equations is a long and arduous process. To solve this problem, the pedographic, anthropometric and tensometric study of the club feet was conducted and the database of patients was created. Statistical analysis of the database patients was carried out. The results of statistical analysis of the database were used to carry out the design of the main section and the orthopedic boot tree print. Taking into account the biomechanics of movement of the foot the obtained parameters are transformed and curvilinear lines of the surface of the boot tree are determined. One of the main problems in the process of design and manufacturing of the orthopedic boot trees is to develop a new mathematical algorithm for geometric surface of the boot tree.

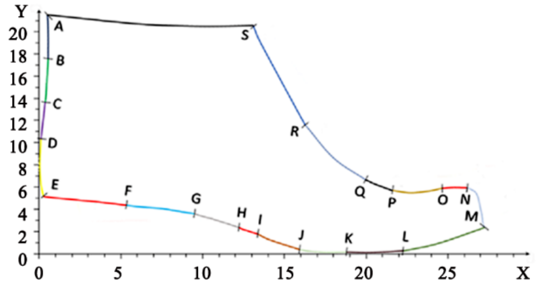

Fig. 1. The longitudinal-vertical section of orthopedic boot tree.

The algorithm describing the geometrical shape of the surface of boot tree was considered in [1-5]. The researchers used the following methods of mathematical investigation: radius-graphical, bi-quadratic spline, bi-cubic interpolating spline for describing the transverse-vertical, longitudinal-horizontal sections and footprint. Those methods were quite complex and time-consuming for designing boot trees. Also, they are characterized by certain inaccuracies. For system description of the transverse-vertical sections, the integral curves of the solutions to singular Dirichlet boundary value problem were studied and used. Using the same method (the integral curves of the solutions to singular Dirichlet boundary value problem), we constructed the transverse-vertical section orthopedic boot tree. Fig.1 illustrates the longitudinal-vertical section of orthopedic boot tree. To obtain a shape of this section, we used mainly a database of patients. In order to construct the longitudinal-vertical section of orthopedic boot tree, we divided it into nineteen parts, and then we made description of each enumerated section on the basis of differential equations below. From the integral curves of the solutions to singular Dirichlet boundary value problem, we chose those nineteen parts that are identical to the basic longitudinal-vertical geometrical shapes of orthopedic boot tree, in particular:

1. AB $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=0 conforms to a set $[12;3,1]\times[10;13,5]$;
2. BC $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=2 conforms to a set $[2.4;2.85]\times[10;13.5]$;
3. CD $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=5, conforms to a set $[3,4;3.5]\times[13,1;15]$;
4. DE $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=0, conforms to a set $[-2,6;-2,1]\times[-6;-1,9]$;
5. EF $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=3, conforms to a set $[3,1;3,4]\times[8,05;13,1]$;
6. FG $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=3, conforms to a set $[-0,5;-0,4]\times[4,5;5,1]$;
7. GH $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=0, conforms to a set $[2;2,4]\times[1,7;4,1]$;
8. HI $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=2, conforms to a set $[11;12,3]\times[2,8;2,6]$;
9. IJ $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=2, conforms to a set $[-2,5;-2,1]\times[-6;-9]$;
10. JK $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=2, conforms to a set $[-4;0]\times[0,6;1]$;
11. KL $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=4, conforms to a set $[-2,5;-2]\times[2;5,3]$;
12. LM $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=0, conforms to a set $[-3,2;-2,6]\times[-12,5;-6,1]$;
13. MN $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=3, conforms to a set $[0,5;0,8]\times[7,5;11]$;
14. NO $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=1, conforms to a set $[-2,5;-2,3]\times[5,1;7,5]$;
15. OP $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=2, conforms to a set $[-0,8;-0,55]\times[-7;-3,1]$;
16. PQ $u(t)=(-\frac{1}{3}-\frac{1}{3}c)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, at c=2, conforms to a set $[-2,1;-1,8]\times[0,1;1,5]$;

17. QR $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=0, conforms to a set $\left[1,7;2,4\right]\times[1,1;4,9]$;

18. RS $u(t)=\frac{t^3}{2}+\frac{1}{3}ct^{-2}-(1-\frac{1}{3}c)t+\frac{1}{2}$, at c=0, conforms to a set $\left[-0,9;0\right]\times[1,1;0,55]$;

19. a set SA $\alpha=\ell^{\alpha}\cos\beta$ $\varphi(x)=2\cdot x+1$ [-3,45;-0,94][4,4;5,97]. By means of computer program, we executed turning and connection of the curves, on the basis of which the main longitudinal-vertical section of orthopedic boot tree shown in Fig.1 was obtained.

Construction of the front of the boot tree print with high accuracy was quite difficult. To solve this problem, the differential equations with deviating argument were used. The papers [9-10] deal with the equation with “delayed” argument, as well as the differential equation with "advanced" argument. The same papers describe the solutions to the differential equations with deviating argument in the form of curves.

By means of the above mentioned methods (the integral curves of the solutions to singular Dirichlet boundary value problem and the equations with deviating argument), we constructed the orthopedic boot tree print. Fig.2 illustrates the orthopedic boot tree print.

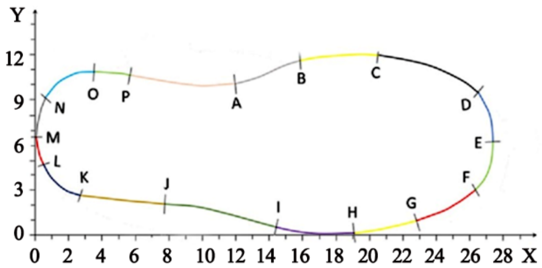

Fig. 2. The orthopedic boot tree print.

To obtain a shape of the section shown in Fig.2, the database of patients was used. In order to construct the orthopedic boot tree print, we divided it previously into sixteen parts, and then we made description of each enumerated section on the basis of differential equations.

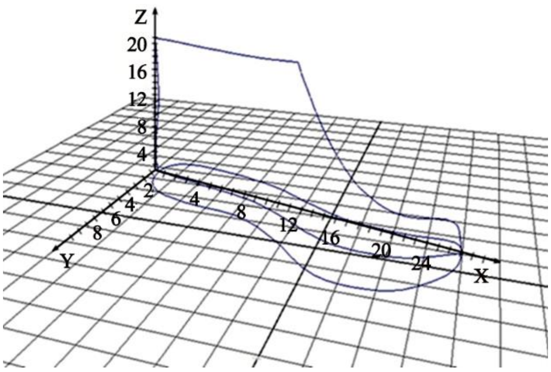

Fig. 3. The main longitudinal-vertical section and print in spatial format.

Using the program of 3D design (Delcam), Fig.3 illustrates the main longitudinal-vertical section and print in spatial format.

Thus, based on a database of patients the longitudinal-vertical section and the shapes of the print of orthopedic boot tree, were constructed by means of the integral curves of the solution to singular Dirichlet boundary value problem and the curves of the solutions to the differential equations with deviating argument was constructed. This method allows description of the longitudinal-vertical sections and the shapes of the orthopedic boot tree print with high accuracy. It also enables us to change the longitudinal-vertical sections and the shapes of the print of orthopedic boot tree in unlimited number of times while changing the sizes of orthopedic boot trees. The latter is particularly relevant for manufacturing of orthopedic shoes, when we deal with patients having the nonstandard or club feet.

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მექანიკა

ორთოპედიული ფეხსაცმლის კალაპოტის კვალისა და ძირითადი გრძივი-ვერტიკალური კვეთის აგება დიფერენციალური განტოლებების ამონახსნების წირების საშუალებით

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**აკაკი წერეთლის სახელმწიფო უნივერსიტეტი, დიზაინისა და ტექნოლოგიის დეპარტამენტი, ქუთაისი, საქართველო*
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(წარმოდგენილია აკადემიის წევრის გ. გაბრიჩიძის მიერ)

სტატიაში განხილულია ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი გრძივი-ვერტიკალური კვეთისა და კვალის აგების მეთოდები და კომპიუტერული პროექტირების საკითხები. აღნიშნული საკითხის გადასაჭრელად სტატიის ავტორთა ჯგუფი ძირითადად ეყრდნობოდა პაციენტთა მონაცემების ბაზას, რომელშიც წარმოდგენილი იყო დეფორმირებული ტერფების ანთროპომეტრიული, პედოგრაფიული და ტენზომეტრული მონაცემები. ორთოპედიული ფეხსაცმლის დასამზადებლად საჭიროა ისეთი კალაპოტების პროექტირება, სადაც აუცილებლად უნდა იყოს გათვალისწინებული დეფორმირებული ტერფის პათოლოგიური გადახრები. დეფორმირებული ტერფის ნორმალური ფუნქციონირებისათვის საჭიროა შევიმუშავოთ ფეხსაცმლის ისეთი შიგა ფორმა, რომლის ტარების პროცესში პაციენტი თავს გრძნობდეს კომფორტულად. ორთოპედიული ფეხსაცმლის კალაპოტი გეომეტრიული თვალსაზრისით რთული ფორმისაა და მისი აღწერა მათემატიკური კვლევის მეთოდების გამოყენებით წარმოადგენს საკმაოდ რთულ და შრომატევად პროცესს. სტატიის ავტორთა ჯგუფის მიერ აღნიშნული პრობლემის გადასაწყვეტად გამოყენებულია დირიხლეს სინგულარული სასაზღვრო ამოცანისა და გადახრილარგუმენტიანი განტოლებების ამონახსნთა ინტეგრალური წირები ორთოპედიული კალაპოტის ძირითადი გრძივი-ვერტიკალური კვეთისა და კვალის ასაგებად. კომპიუტერული პროგრამების გამოყენებით ვაწარმოებდით მიღებული წირების მონაკვეთების შეუღლებასა და გადაბმას, რის საფუძველზეც აგებულია ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი გრძივი-ვერტიკალური კვეთი და კალაპოტის კვალი. სტატიაში წარმოდგენილია ასევე ძირითადი გრძივი-ვერტიკალური კვეთი და კალაპოტის კვალი 3D ფორმატში.

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