საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 12, №3, 2018
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 12, no.3, 2018

Mechanics

Construction of the Orthopedic Shoe Tree Main Transverse-Vertical Cross-Sections by Means of the Integral Curves

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(Presented by Academy Member Guram Gabrichidze)

ABSTRACT. The paper dwells on the design of such shoe trees for producing the orthopedic shoes for patients with club and abnormal foot, in which the internal shape of the shoe required for the normal functioning of each abnormal foot is envisaged. The article also describes the problem of constructing the shapes of transverse-vertical cross-sections of the orthopedic shoe trees (of the main cross sections 0.72D, 0.62D and 0.18D). In the footwear industry, considerable attention is paid to the issue of boot tree design. The task is much more difficult, when we deal with design and production of the orthopedic shoe tree. It is well-known that orthopedic boot tree is of very complex form and its description using the mathematical methods of research is a labor-intensive process. For the purpose of solving the mentioned problem, that is, for the construction of the shapes of transverse-vertical cross-sections of the orthopedic shoe tree, the team of authors used the integral curves of the solutions of the differential equations. By articulation and binding of these curves, there have been constructed the main shapes of the orthopedic shoe tree (0.72D, 0.62D and 0.18D) with high accuracy (exact counterpart of patterns of the main cross-sections of boot tree have been obtained). This method also allows for changing the transverse-vertical cross-sections of boot trees in an infinite number, when transferring from one size to another one. The latter is of particular relevance in the process of design and production of the orthopedic shoe tress. This method also allows for changing the transverse-vertical cross-sections of boot trees in an infinite number, when transferring from one size to another one. The latter is of particular relevance in the process of design and production of the orthopedic shoe trees. *©2018 Bull. Georg. Natl. Acad. Sci.*

Key words: orthopedic shoe boot tree, integral curves

The team of the authors conducted the pedographic, anthropometric and tensometric studies of club feet. The patient database was created, in which pathological abnormalities of foot of each patient are individually registered, which are the basic baseline data to design and produce boot trees for the orthopedic shoes intended for people the group.

To produce the orthopedic shoes for patients with club and abnormal feet, it is necessary to design such boot trees, in which the internal shape of the shoe required for the normal functioning of each abnormal foot is envisaged. Therefore, the basic sizes and shapes of boot tree should be consistent with the sizes and shapes of the abnormal feet.

In the process of pedographic, anthropometric and tensometric measurements of patients with club and abnormal feet, the following data were registered:

Foot sizes in static position;

Foot size changes in the dynamics;

Loads coming on the local sections of foot;

The relationships between the foot sizes and shapes.

Based on the anthropometric, tensometric and pedographic measurements, a database of patients with club and abnormal feet was created, in which the results of these study were registered.

The above-mentioned integrated approaches allow us to produce the comfortable orthopedic shoes. The prerequisite for the normal functioning of club and abnormal feet consists in the fact that foot should be placed in the shoe conveniently. Consequently, the process of designing and producing of orthopedic boot trees should envisage the loads coming on a plantar area of foot, which must be in line with the shape of the supporting plane of the shoe so that the loads are evenly distributed on the contact area.

The following data should be taken into account in the process of transition from the shapes and sizes of club and abnormal feet to the shapes and sizes of orthopedic boot trees:

Foot size changes in the dynamics;

Digital features of the foot size changes on certain sections of foot;

Foot size and shape changes in the footprint and the toe bone articulation;

Foot size changes in the transverse-vertical cross-sections.

During the process of human movement, all changes in the feet and sizes and shapes (in relation to the initial condition) should be taken into account in the process of designing the orthopedic shoe boot tree.

Proceeding from the bio-mechanical characteristics of foot, we should transform the size parameters, and on their basis, we should determine the integral curves of the boot tree surface. Development of a new algorithm describing the geometric surface of boot tree and its use in practice is one of the main and pressing problems in the orthopedic shoe boot tree industry. The algorithm describing the geometric shape of a surface of boot tree is considered in the works of researchers [1-5]. For describing the geometric shape (transverse-vertical and longitudinal horizontal cross-sections) of the orthopedic shoe boot tree, they used the following methods of research: radius-graphical, biquadratic spline, and bicubic interpolating spline. These methods are very complex and time-consuming during the design process, and are characterized by certain inaccuracies. For the system description of the transverse-vertical cross-sections, the team of authors of the article studied the integral curves of the solutions of the singular boundary Dirichlet problem, particularly, the article [6] dwells on the singular boundary Dirichlet problem:

$$u''(t)+\frac{a}{t}\cdot u'(t)-\frac{a}{t^2}u(t)=f(t,u(t),u'(t)) \tag{1}$$

$$u(t)=0 \qquad u(T)=0\,, \tag{2}$$

where $a\in(-\infty;1)$, f satisfies Caratheodory's local condition for a set $[0,T]\times D$ $D=(0,\infty\times R)$. In the article [7], I. Rachunkova, A. Spielaurer, S. Stanek and E. B. Weinmuler studied the issue of the existence of solutions to (1)-(2) problems, and the solution to (1)-(2) problems is obtained explicitly:

$$u(t)=c_1 t+c_2 t^{-a}+t\int_t^T S^{-a-2}\left(\int_S^t \xi^{a+1} f(t,u(\xi),u'(\xi)d\xi\right)d\xi\ , \tag{3}$$

where $c_1,\ c_2 \in R \quad t\in[0,T]$.

In the article [8], we consider two special cases of (1)-(2) problems,

I. $$u''-\frac{2}{t}u'-\frac{2}{t^2}u=t \tag{4}$$

$$u(1)=0\ \ u'(1)=c\,. \tag{5}$$

The solution to (4)-(5) problem is:

$$u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}.$$

The integral curves of these solutions are constructed, when c=0;1;2;3;4;5. (Fig. 1).

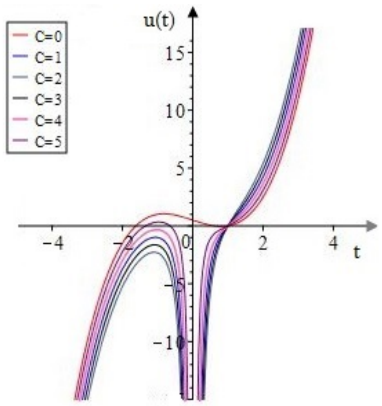

Fig. 1. The integral curves of the solution to (4)-(5) problems, when c=0;1;2;3;4;5.

II. $$u''+\frac{a}{t}u'-\frac{a}{t^2}u=t^2 \tag{6}$$

$$u(1)=0\ \ u'(1)=c\,. \tag{7}$$

The solution to (6)-(7) problem is:

$$u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}.$$

The integral curves of these solutions are constructed, when c=0;1;2;3;4;5. (Fig. 2).

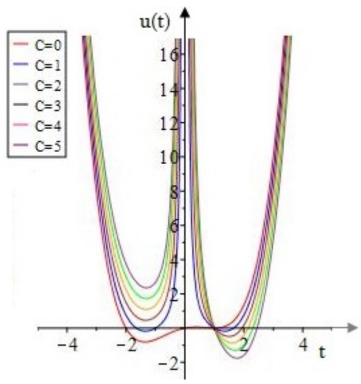

Fig. 2. The integral curves of the solution to (6)-(7) problem, when c=0;1;2;3;4;5.

By means of the constructed curves, we can describe to a high accuracy the transverse-vertical cross-sections of the ortyhopedic shoe boot tree. By variation of the constant c, it is possible to determine the shape of any transverse-vertical cross-section of boot tree, as well as it is possible to change the shapes of the cross-section, when transferring from one size of boot tree to another one. The use of the mentioned mathematical algorithm is relevant during the process of designing the orthopedic shoe boot tree, when we deal with patients having club and abnormal feet.

During the process of practical implementation of the above mentioned method, there ws used a database of patients with club and abnormal feet, which was created by the team of authors of this article, and in which the results of the pedographic, anthropometric and tensometric studies are registered, and for a comparative analysis, there have been taken the patterns of the transverse-vertical cross-section of the existing orthopedic shoe boot tree (women's orthopedic shoe boot tree of size 38).
Fig. 3 illustrates the transverse-vertical cross-section of the orthopedic shoe boot tree on 0.72D (D – is a foot length). In order to obtain the mentioned shape, we mostly used a database of patients and the patterns of the transverse-vertical cross-section of the existing orthopedic shoe boot tree. Based on this, we previously divided into five parts the shape of the pattern's transverse-vertical cross-section on 0.72D.

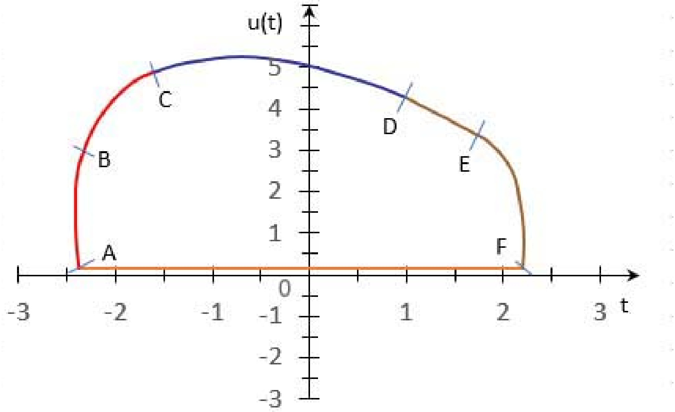

Fig. 3. Transverse-vertical cross-section of boot tree on 0.72D.

From the integral curves we constructed (Fig1, Fig. 2), we select the geometric shapes identical (corresponding) to these five curves - AB curve, BC curve, CD curve, DE curve and EF curve, in particular:

for the AB curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $[-1;-0.6]\times[2.6;5]$ for c=1;

for the BC curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$ which corresponds with a set $[-1;-0.4]\times[0;1.6]$ for c=3;

for the CD curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $[-2.8;-1.75]\times[6.8;0]$ for c=1;

for the DE curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-1;0\right]\times[1.2;0.5]$, for c=0;

for the EF curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $\left[-2.2;-1.4\right]\times[1.5;-0.8]$, for c=0.

The curve of a function $u(t)=\frac{1}{6}$ corresponds with the AF curve (the lower part) of 0.72D cross-section of boot tree.

With a computer program, by turning and parallel translation of the mentioned curves, the transverse-vertical cross-section on 0.72D was constructed, which is an exact counterpart of pattern.

Likewise, we constructed the transverse-vertical cross-sections of the orthopedic shoe boot tree on 0.62D and 0.18D. On 0.62D, we divided the transverse-vertical cross-section into eight parts: AB curve, BC curve, CD curve, DE curve, EF curve, FP curve, PQ curve and QA curve. (Fig. 4).

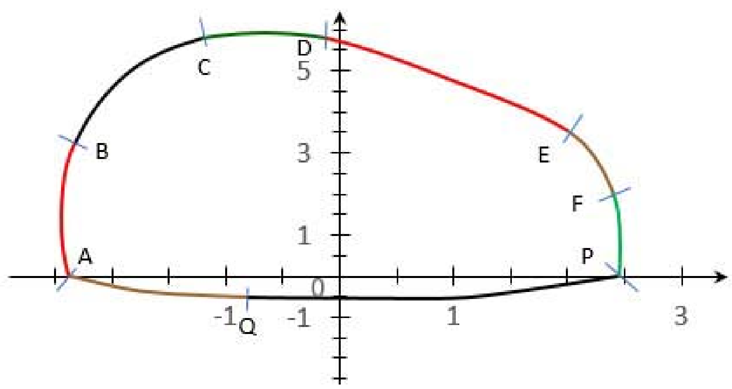

Fig. 4. Transverse-vertical cross-section of boot tree on 0.62D.

Similarly to 0.72D cross-section, from the integral curves on 0.62D cross-section we select the geometric shapes identical (corresponding) to these eight curves (Fig. 1, Fig. 2), in particular:

for the AB curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$ which corresponds with a set $\left[-2.7;-1.9\right]\times[3;-0.4]$, for c=0;

for the BC, curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-2.2;-0.8\right]\times[-2.1;1]$, for c=0;

for the CD curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-0.7;-0.5\right]\times[-3.6;-7.5]$, for c=2;

for the DE curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $\left[-1.95;-2.65\right]\times[-1.4;-7.3]$, for c=0;

for the EF curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $[-1.8;-2.5]\times[-1.8;-0.4]$, for c=3;

for the FP curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $[-2.1;-1.6]\times[-5.8;-3]$, for c=2;

for the PQ curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $[-3;-2.8]\times[13;9.1]$, for c=3;

for the QA curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $[0.4;0.8]\times[7;0.6]$, for c=3.

Here again, with a computer program, by turning and parallel translation of the mentioned curves, the transverse-vertical cross-section on 0.62D (Fig. 4) was constructed, which is an exact counterpart of pattern. On 0.18D, we divided the transverse-vertical cross-section into ten parts: AB curve, BC curve, CD curve, DE curve, EF curve, FP curve, PQ curve, QM curve, MN curve and NA curve. (Fig. 5).

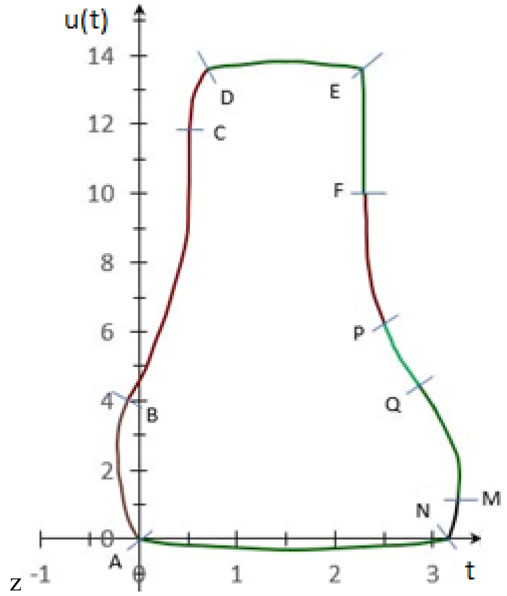

Fig.5. Transverse-vertical cross-section of boot tree on 0.18D.

From the integral curves shown in Fig. 1 and Fig. 2, we select the geometric shapes identical (corresponding) to these ten curves, in particular:

for the AB curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $[-2.6;-1.6]\times[4.05;-0.6]$, for c=5;

for the BC curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $[1.56;2.85]\times[0.8;11.8]$, for c=0;

for the CD curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-0.8;0.8\right]\times[0.8;0.3]$, for c=0;

for the DE curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-7.5;-1.3\right]\times[0.5;0.8]$, for c=2;

for the EF curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-0.5;-0.8\right]\times[-8;-12.6]$, for c=3;

for the FP curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $\left[-2.5;-2\right]\times[3;-0.4]$, for c=0;

for the PQ curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $\left[-2.5;-2\right]\times[4.8;3]$, for c=2;

for the QM curve, we take that part of the solution to $u(t)=\left(-\frac{1}{3}-\frac{1}{3}c\right)t+\frac{1}{3}c\frac{1}{t^2}+\frac{2}{3}t+\frac{t^2}{2}+\frac{t^4}{6}$, which corresponds with a set $\left[-0.4;0\right]\times[-0.85;0.6]$, for c=0;

for the MN curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[-0.5;-0.25\right]\times[-3;-0.85]$, for c=5;

for the NA curve, we take that part of the solution to $u(t)=\frac{t^3}{2}-\frac{1}{3}ct^{-2}-\left(1-\frac{1}{3}c\right)t+\frac{1}{2}$, which corresponds with a set $\left[1.5;2.85\right]\times[1.7;14.85]$, for c=2.

With a computer program, by turning and parallel translation of the mentioned curves, the transverse-vertical cross-section on 0.18D (Fig. 4) was constructed, which is an exact counterpart of pattern.

Hence, by means of the integral curves of the solutions of the singular boundary Dirichlet problem, according to the database of patients and existing patterns of the orthopedic shoe boot tree (for a comparative analysis) the team of authors of the article constructed the shapes of the main 0.72D, 0.62D and 0.18D transverse-horizontal cross-sections of boot tree. This method allows us to describe with high accuracy the transverse-vertical shapes of boot tree (the results are the exact counterparts of the patterns of main cross-sections of the existing orthopedic shoe boot tree). It also enables the cross-sectional vertical shapes to fill in an unlimited amount of time from one level to the other. The latter is particularly relevant in the production of orthopedic shoe when it comes to patients with nonstandard or deformed and abnormal feet.

The work was fulfilled with the financial support of Shota Rustaveli National Science Foundation, Grant FR № 217386.

მექანიკა

ორთოპედიული ფეხსაცმლის კალაპოტის ძირითადი განივ-ვერტიკალური კვეთების აგება ინტეგრალური წირების საშუალებით

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სტატიაში განხილულია დეფორმირებული და პათოლოგიური ტერფების მქონე პაციენტებისათვის ორთოპედიული ფეხსაცმლის დასამზადებლად ისეთი კალაპოტების პროექტირება, სადაც მაქსიმალურად იქნება გათვალისწინებული თითოეული დეფორმირებული და პათოლოგიური ტერფის ნორმალური ფუნქციონირებისათვის საჭირო ფეხსაცმლის შიგა ფორმა. განხილულია ორთოპედიული ფეხსაცმლის კალაპოტების განივ-ვერტიკალური კვეთების (ძირითადი კვეთების **0,72D**, **0,62D** და **0,18D**) ფორმების აგების პრობლემა. ფეხსაცმლის მრეწველობაში დიდი ყურადღება ეთმობა კალაპოტების პროექტირების საკითხს. ცნობილია, რომ ფეხსაცმლის კალაპოტი არის რთული ფორმის და მისი აღწერა მათემატიკური კვლევის მეთოდების გამოყენებით წარმოადგენს საკმაოდ შრომატევად პროცესს. სტატიის ავტორთა ჯგუფის მიერ აღნიშნული პრობლემის გადაწყვეტის მიზნით, ანუ ორთოპედიული ფეხსაცმლის კალაპოტის განივ-ვერტიკალური კვეთების ფორმების აგებისათვის გამოყენებულია დიფერენციალურ განტოლებათა ამონახსნების ინტეგრალური წირები. აღნიშნული წირების მონაკვეთების შეუღლებითა და გადაბმით აგებულია ორთოპედიული ფეხსაცმლის კალაპოტის განივ-ვერტიკალური კვეთების (**0,72D**, **0,62D** და **0,18D**) ძირითადი ფორმები დიდი სიზუსტით (მიღებულია კალაპოტის ძირითადი კვეთების შაბლონების ზუსტი ანალოგი). აღნიშნული მეთოდი, ასევე იძლევა საშუალებას კალაპოტის ერთი ზომიდან მეორე ზომაზე გადასვლის დროს, განუსაზღვრელი რაოდენობით და დიდი სიზუსტით ვცვალოთ კალაპოტების განივ-ვერტიკალური კვეთების ფორმები. ეს უკანასკნელი განსაკუთრებით აქტუალურია ორთოპედიული ფეხსაცმლის კალაპოტების პროექტირებისა და წარმოების პროცესში.

REFERENCERS

1. Fukin V.A. (1980) Radiusograficheskii metod konstruirovaniia vnutrennei formy obuvi. 381 s. M. (in Russian).
2. Fukin V.A., Kostyleva V.V., Lyba V.P. (1987) Proektirovanie obuvnykh kolodok. 268 s. M. (in Russian).
3. Zamarashkin K.N. (2004) Matematicheskie metody proektirovaniia obuvi i konstruirovaniia tekhnologicheskoi osnastki. SPB. SPGUTD, s. 312.
4. Fukin V.A. (2000) Teoreticheskie osnovy proektirovaniia vnutrennei formy obuvi. 2-izdanie per. i dop.s. 356. M. (in Russian).
5. Kiselev S.Yu. (2004) Avtomatizirovannoe proektirovanie i izgotovlenie tekhnologicheskoi osnastki dlia proizvodstva obuvi i protezo-ortopedicheskikh izdelii. Dokt. Diss. 392 s. M. (in Russian).
6. Shalamberidze M., Sokhadze Z., Tatvidze M. (2018) Construction of the transverse-vertical shapes of the orthopedic boot-tree by means of the solution to singular dirichlet boundary value problem. *Bull. Georg. Natl. Acad. Sci*., 1: 27-32.
7. Rachunkova I., Spielaurer A., Stanek S. and Weinmuler E. B. (2013) The structure of a set of positive solutions to Dirichlet BVPs with time and space singularities. *Georgian Mathematical Journal*, 1: 95-127.

Received April, 2018